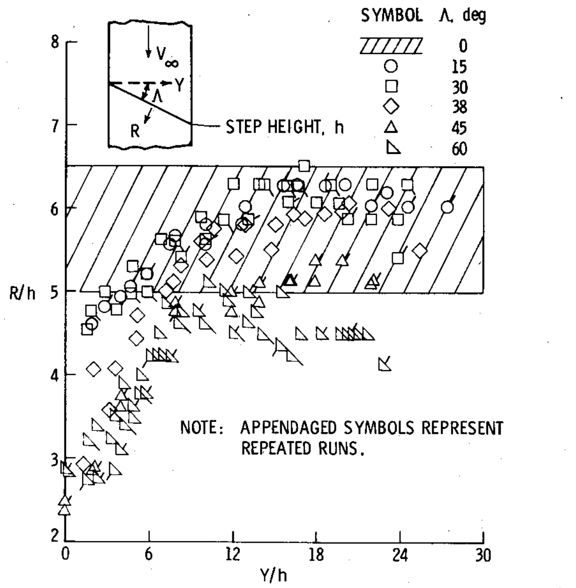

Fig. 2 Reattachment distance vs spanwise coordinate for various sweep angles ($h = 1.27$ cm; hatched area represents two-dimensional results).

viscosity of the coating oil was too high, the droplets would remain stationary. Reattachment distance measurements obtained in this manner were repeatable within ± 0.5 step heights. A typical oil flow pattern produced in the manner described is shown in Fig. 1. Additional photographs are presented in Ref. 11.

Reattachment distances measured in planes normal to the step face for $h = 1.27$ cm are displayed in Fig. 2 as a function of the spanwise coordinate. From these data it is possible to determine the range of sweep angles over which the "independence principle" applies (and beyond which sweep effects dominate) by identifying when asymptotic values of the reattachment distance fail to coincide with similar data at lower sweep angles. From Fig. 2 it can be observed that for $\Lambda > 38$ deg ($\approx$"Λ_{crit}") the reattachment distance data fail to follow the trend of the data at smaller sweep angles and generally fall outside the reattachment region (reattachment point fluctuates in the streamwise direction) defined by $\Lambda = 0$ deg. With $h = 0.32$ and 0.79 cm, the effect of sweep is not as great, with the result that $\Lambda_{crit} > 38$ deg. At $h = 2.38$ cm, there is insufficient span for asymptotic values of R to be reached, so the data are inconclusive.

It can be concluded that the "independence principle" is valid up to $\Lambda \approx 38$ deg for $h < 1.27$ cm. The validity of this principle allows the application of two-dimensional analyses (in the proper coordinate system) to the separated flow associated with swept steps for $\Lambda < 38$ deg and $h/\delta < 1$.

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Comparison Between Navier-Stokes and Thin-Layer Computations for Separated Supersonic Flow

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Introduction

IN the numerical simulation of high Reynolds-number flow, one can frequently supply only enough grid points to resolve the viscous terms in a thin layer. As a consequence, a body- or stream-aligned coordinate system is frequently used and viscous terms in this direction are discarded. It is argued that these terms cannot be resolved and computational efficiency is gained by their neglect. Dropping the streamwise viscous terms in this manner has been termed the thin-layer approximation. The thin-layer concept is an old one, and similar viscous terms are dropped, for example, in parabolized Navier-Stokes schemes. However, such schemes also make additional assumptions so that the equations can be marched in space, and such a restriction is not usually imposed on a thin-layer model.

The thin-layer approximation can be justified in much the same way as the boundary-layer approximation; it requires, therefore, a body- or stream-aligned coordinate and a high Reynolds number. Unlike the boundary-layer approximation, the same equations are used throughout, so there is no matching problem. Furthermore, the normal momentum equation is not simplified and the convection terms are not one-sided differenced for marching. Consequently, the thin-layer equations are numerically well behaved at separation and require no special treatment there.

Nevertheless, the thin-layer approximation receives criticism. It has been suggested that the approximation is invalid at separation and, more recently, that it is inadequate for unsteady transonic flow.[1] Although previous comparisons between the thin-layer and Navier-Stokes equations have been made, these comparisons have not been adequately documented.

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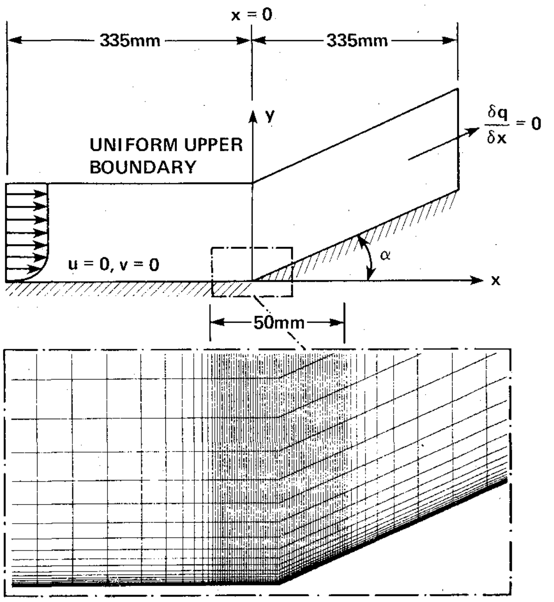

Fig. 1 Computational domain and grid.

In the present work we describe a detailed comparison between the thin-layer and Navier-Stokes equations for supersonic flow over a ramp (the geometry and a portion of the finite difference grid are shown in Fig. 1). This flow was selected because it is separated and because the strong interaction region is confined to the ramp juncture. Consequently, good streamwise grid resolution can be supplied for the small separation region. Therefore, if the streamwise viscous terms are important, they can be computed with a much finer streamwise grid spacing than usual.

Although experimental data and other simulations are available for this case,[2] the interest here is a comparison between the thin-layer and the Navier-Stokes equations. Consequently, we use the same numerical method, grid, and turbulence model in both simulations. Unsteady effects are compared by moving the ramp and tracing the solution in time. The results of this study as described below confirm the validity of the thin-layer model for this class of problems.

Governing Equations and Numerical Algorithms

The strong conservation law form of the Navier-Stokes equations in general coordinates can be written as[3,5]

$$\frac{\partial \hat{q}}{\partial t}+\frac{\partial \hat{E}}{\partial \xi}+\frac{\partial \hat{F}}{\partial \eta}=Re^{-1}\left(\frac{\partial \hat{R}}{\partial \xi}+\frac{\partial \hat{S}}{\partial \eta}\right) \tag{1}$$

where

$$\hat{R}=\hat{R}^{\xi}(\bar{q},\bar{q}_{\xi})+\hat{R}^{\eta}(\bar{q},\bar{q}_{\eta})=(\xi_x\bar{R}+\xi_y\bar{S})/J \tag{2}$$

and

$$\hat{S}=\hat{S}^{\xi}(\bar{q},\bar{q}_{\xi})+\hat{S}^{\eta}(\bar{q},\bar{q}_{\eta})=(\eta_x\bar{R}+\eta_y\bar{S})/J \tag{3}$$

For the thin-layer approximation, all of the viscous terms containing ξ derivatives are neglected (i.e., $\hat{R}^{\xi}$, $\hat{R}^{\eta}$, and $\hat{S}^{\xi}$) and only the $\hat{S}^{\eta}$ term is retained. An implicit approximate factorization finite difference algorithm is used here to solve Eq. (1). Details can be found in Refs. 3-5.

The Sutherland law is used to calculate the laminar viscosity, the effects of turbulence are simulated in terms of an eddy viscosity coefficient, and the algebraic model of Baldwin and Lomax[6] is used.

Results

Computed results were obtained for both the Navier-Stokes and the thin-layer equations using the previously defined transformed equations and finite difference algorithms. The 120×48 grid of which a portion is shown in Fig. 1, was used for all the calculations ($\Delta x/\delta_0=0.045$ in the bubble, where δ_0 is the boundary-layer thickness upstream of the bubble). To obtain the initial profile, a run was first made with undisturbed flow upstream of a finite flat plate, the length of the plate being chosen to set the Reynolds number from experiment ($Re_\infty=63\times10^6$/m).[2] A freestream Mach number of 2.85 was used and runs were made for ramp angles of 20 and 24 deg. (Only the 20-deg results are shown, as similar results were obtained for both ramp angles.)

Steady-state computed and experimental pressure distributions, skin friction, and velocity profiles (taken at the

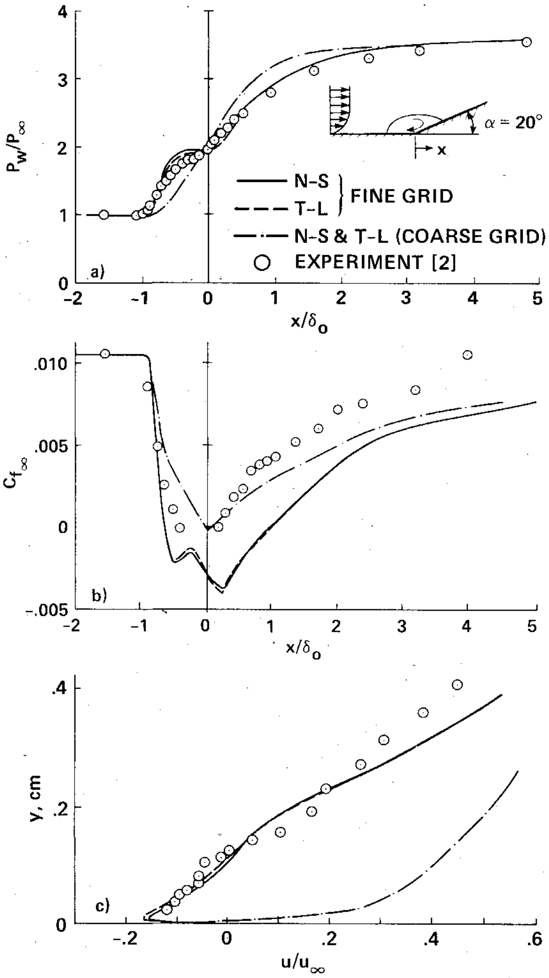

Fig. 2 Comparison of computation and experiment, $\alpha=20$ deg, $M_\infty=2.85$, $Re_{\delta_0}=1.65\times10^6$. a) Surface pressure. b) Skin friction. c) Computational and experimental velocity profiles at $x=0$.

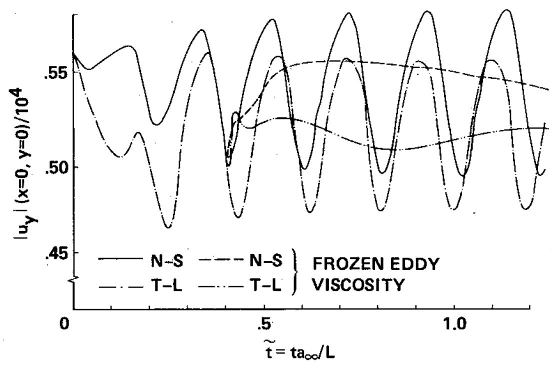

Fig. 3 Time history of $|\partial u/\partial y|$ at $x=0$ and $y=0$ after a disturbance (fixed ramp).

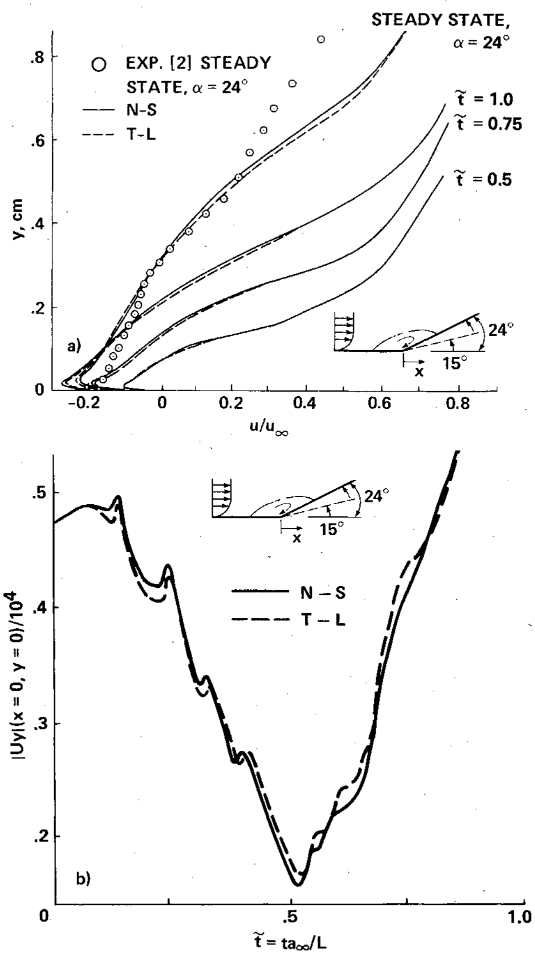

Fig. 4 Comparison of theoretical results for a moving ramp. a) Velocity profiles at various times at $x=0$. b) Time tracing of $|\partial u/\partial y|$ at $x=0$ and $y=0$.

ramp-plate intersection) are shown in Fig. 2. As shown, one does indeed detect small differences between the thin-layer and Navier-Stokes results, but the discrepancies are minor and are confined to the separation bubble region only. Results are also shown for both the Navier-Stokes and the thin-layer equations that were obtained using a coarsened grid, 60×48, with uniform Δx, $\Delta x/\delta_0 = 0.62$ (in comparison to 0.045 for the fine grid). While the results for the fine grid are equally good with respect to the experimental data, the results for the coarse grid show no difference between the Navier-Stokes and thin-layer models and show poor agreement with the experiment.

As previously mentioned, the minor discrepancy between the two simulations is confined to the separation bubble only. Examination of the time development of this region shows that a truly steady solution is not only difficult to obtain, but is obtained only after a large number of time steps are taken and if sufficiently high values of the smoothing coefficients are used. This is demonstrated by Fig. 3, which shows the time history of the velocity gradient at the ramp-plate juncture after a small perturbation is imposed on the steady flowfield at $t=0$. Although the difference of mean values between the two viscous models is small, a larger local difference can be detected because of the phase shift. In our view, the discrepancy between the thin-layer and Navier-Stokes equations is as much due to this phase shift as to any other differences. The oscillations shown in Fig. 3 are confined to the separation bubble region only, and are probably caused by interaction between the turbulence model and the flowfield. If the eddy viscosity is frozen at a fixed time ($\tilde{t}=0.4$ in Fig. 3), the solution quickly reaches a much more quiescent state. Strategies to accelerate the solution to a steady state have been less effective in viscous flow than in inviscid flow; this may be due to a turbulence model interaction.

The results of Chyu and Kuwahara[1] suggest that the streamwise viscous terms might be more significant in unsteady flow. To test this on our case we started with a steady Navier-Stokes solution obtained for a 15-deg ramp and generated a nonsteady flow by moving the ramp to a 24-deg angle at a constant velocity of 0.045 deg per nondimensional time step ($\Delta\tilde{t}=0.005$, $\tilde{t}=ta_\infty/L$ where $L=1$ m). Figure 4a shows the velocity profiles at various times and Fig. 4b illustrates a time trace of $|\partial u/\partial y|$ at $x=0$, the plate-ramp juncture, and $y=0$. Small consistent differences are observed, but the overall theoretical results are equivalent, and these differences decrease for smaller Δt. The agreement between experiment and theory as shown in Fig. 4a is comparable to that obtained by Horstman et al.[2] using a similar turbulence model.

Although not shown, it is remarked that if the grid lines intersect the plate at a more skewed angle, the solution results will change. However, the thin-layer and Navier-Stokes results remain in just as good agreement on the skewed grid (up to 20-deg difference from the normal) as they do above.

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